Math 140 — Calculus 1
Measurable Outcomes

Mathematics Department, UMass Boston

Reference text: Numbers in brackets refer to sections of Stewart, *Calculus,* eighth edition Early Transcendentals

Note: Outcomes marked **(Optional)** may appear on the final exam with the unanimous consent of all instructors.

1. Review

1(a) Simplify expressions involving exponents.

1(b) Use logarithms to solve exponential equations.

1(c) Use the properties of logarithms to simplify an expression.

1(d) **(Optional)** Model a periodic phenomenon as a sine wave, determining its amplitude etc.

1(e) Without a calculator, evaluate a^0, $\log_b(1)$, and $\sin\theta$ and $\cos\theta$ for $\theta = k\pi/2$ at least.

2. Slopes and rates of change

2(a) Approximate the slope of $f(x)$ at a point graphically. [2.1]

2(b) Approximate the slope of $f(x)$ at a point using a difference quotient. [2.1]

2(c) Find the average rate of change of a function from a table. [2.1]

2(d) Interpret the slope of a function as a rate of change / sensitivity, including units. [2.1]

3. Limits

3(a) Approximate a limit using a table. [2.2]

3(b) Distinguish between a finite limit, infinite limit, and no limit using a table. [2.2]

3(c) Approximate limits (including one-sided limits) from a graph. [2.2]

3(d) Understand that $\lim_{x \to a} f(x)$ does not depend on $f(a)$. [2.2]

3(e) Determine the value of a limit given information about the corresponding one-sided limits. [2.2]

3(f) **(Optional)** Use the Squeeze Theorem to evaluate a limit. [2.3]

3(g) Find the limit of common functions at ∞ and $-\infty$. [2.6]

3(h) Find the limit of a ratio of functions at ∞ or $-\infty$ using formal algebraic manipulations. [2.6]

3(i) Find the limit of a ratio of functions at ∞ or $-\infty$ using informal reasoning about "lower-order terms". [2.6]

4. Continuity

4(a) Evaluate a limit using continuity. [2.3, 2.5]

4(b) Evaluate the limit of a rational function by simplifying and then using continuity. [2.3, 2.5]

4(c) Evaluate one-sided limits of a piecewise continuous function using continuity. [2.3, 2.5]

4(d) Given that $f(x)$ satisfies certain conditions, determine what other conditions are necessary in order for $f(x)$ to be continuous. [2.5]

4(e) Determine whether a given piecewise function is continuous at a point. [2.5]

4(f) Determine the location and type of discontinuities of $f(x)$ on $[a, b]$. [2.5]

4(g) Use the Intermediate Value Theorem to write an argument that proves that a function has a root on a given interval. [2.5]

4(h) Critique a given argument that uses the Intermediate Value Theorem. [2.5]

4(i) **(Optional)** Use the Bisection Method and the Intermediate Value Theorem to approximate the root of a function to a given degree of accuracy. [2.5]

5. The Derivative

5(a) Evaluate the derivative of a function at a point using the limit definition. [2.7]

5(b) Sketch a rough graph of $f'(x)$ given a graph of $f(x)$. [2.8]

5(c) Determine at which points a function is not differentiable, given its graph. [2.8]

5(d) Interpret the meaning of $f'(a)$ as a rate of change / sensitivity. [2.7, 2.8]

6. Derivative formulas

6(a) Distinguish constants from non-constants. [3.1]

6(b) Distinguish exponential functions from power functions. [3.1]

6(c) Rewrite functions as powers of x when appropriate ($1/x$, $\sqrt{x}$). [3.1]

6(d) Implicitly use linearity when taking derivatives. [3.1]

6(e) Take the derivatives of polynomials and power functions. [3.1]

6(f) Take the derivative of exponential functions. [3.1]

6(g) Take the derivative of trigonometric functions. [3.3]

6(h) Take the derivative of logarithmic functions. [3.6]

6(i) Use the product rule when appropriate. [3.2]

6(j) Use the quotient rule when appropriate. [3.2]

6(k) Distinguish between function composition and function multiplication. [3.4]

6(l) Use the chain rule when appropriate. [3.4]

6(m) Use implicit differentiation when appropriate. [3.5]

6(n) **(Optional)** Use logarithmic differentiation to find the derivative of $f(x)^{g(x)}$. [3.6]

7. Applications of Derivatives

7(a) Given the position function of an object, determine: [3.7]

- Its velocity
- Its acceleration
- When the object is moving up/down (forward/backward)
- When the object is speeding up
- The net distance traveled over a time interval
- The total distance traveled over a time interval

7(b) Solve an abstract related rates problem. (e.g. If $y = f(x)$, find dy/dt when $x = 3$ and $dx/dt = 5$.) [3.9]

7(c) Model a physical situation as a related rates problem. [3.9]

7(d) Use the derivative to write the equation of a tangent line. [3.10]

7(e) Use the tangent line / linearization to approximate a function's value at another point. [3.10]

7(f) **(Optional)** Use differentials to approximate how an error in x affects the error in $f(x)$. [3.10]

7(g) Write a logical argument that uses Rolle's Theorem to prove that a function has at most one root on a given interval. [4.2]

7(h) **(Optional)** Critique a flawed logical argument that misuses Rolle's Theorem. [4.2]

7(i) Use the Mean Value Theorem to bound $f(b)$ given information about $f(a)$ and about $f'(x)$ on $[a, b]$. [4.2]

7(j) Model a physical situation in a way that it generates a problem like the above. [4.2]

8. Optimization

8(a) Find the local and absolute maxima and minima of a function from its graph. [4.1]

8(b) Determine the critical points of a function. [4.1]

8(c) Find the absolute maximum and minimum of a continuous function on a closed interval. [4.1]

8(d) Solve an abstract optimization problem with a two variable function and a constraint. [4.7]

8(e) Model and solve a physical optimization problem with a two variable function and constraint. [4.7]

9. Graphing with calculus

9(a) Determine where a graph is increasing / decreasing by analyzing the sign of $f'(x)$. [4.3]

9(b) Determine where a graph is concave up / concave down by analyzing the sign of $f''(x)$. [4.3]

9(c) Find the inflection points of a function. [4.3]

9(d) Use the First Derivative Test to classify a critical point as a local maximum, local minimum, or neither. [4.3]

9(e) Use the Second Derivative Test to classify a critical point as a local maximum, local minimum, or indeterminate. [4.3]

9(f) Sketch a piece of a graph with a given sign of f' and f''. [4.3]

9(g) Given a graph, determine the intervals where f' is positive and negative. [4.3]

9(h) Given a graph, determine the intervals where f'' is positive and negative. [4.3]

9(i) Sketch the graph of a polynomial using information about f' and f''. [4.5]

9(j) Find the location of the vertical asymptotes of a function. [2.2, 4.5]

9(k) Find the location of the horizontal asymptotes of a function. [2.6, 4.5]

9(l) Sketch the graph of a rational function using asymptotes and information about f'. [4.5]

10. Antiderivatives

10(a) Calculate the antiderivative of a constant. [4.9]

10(b) Calculate the antiderivative of powers of x, including $1/x$. [4.9]

10(c) Calculate the antiderivative of exponential functions. [4.9]

10(d) Calculate the antiderivative of $\sin x$ and $\cos x$. [4.9]

10(e) Implicitly use linearity when calculating antiderivatives. [4.9]

10(f) Solve the initial-value problem $dy/dx = f(x)$, $y(a) = b$. [4.9]

10(g) Find the position of an object given its velocity function and one initial value. [4.9]

10(h) **(Optional)** Find the position of an object given its acceleration function and two initial values. [4.9]

11. Areas, Riemann Sums, and Definite Integrals

11(a) Approximate the area under a curve using a graph and a specified strategy. [5.1]

11(b) Approximate the accumulated change using a table of the rate of change. (e.g. estimate distance traveled given a table of velocity) [5.1]

11(c) Approximate the area under a curve using a Riemann sum, given a formula for $f(x)$. [5.1]

11(d) Evaluate a definite integral of a function using its graph and basic plane geometry (area of triangles, quadrilaterals, portions of circles). [5.2]

11(e) Approximate a definite integral using a Riemann sum with specified parameters. [5.2]

12. Fundamental Theorem of Calculus

12(a) Evaluate a definite integral of a linear combination of basic functions using the Fundamental Theorem of Calculus. [5.3]

12(b) Find the derivative of a function defined in terms of an integral, using the Fundamental Theorem of Calculus. [5.3]

12(c) Evaluate an indefinite integral of a linear combination of basic functions. [5.4]

12(d) Evaluate an indefinite integral of a product or quotient of functions by simplifying the integrand first. [5.4]

12(e) Interpret a definite integral in terms of accumulated change. [5.4]

12(f) Evaluate an indefinite integral in the form $\displaystyle\int cf(g(x)) \cdot g'(x)\, dx$ using a substitution. [5.5]

12(g) Evaluate a definite integral in the form $\displaystyle\int cf(g(x)) \cdot g'(x)\, dx$ using a substitution. [5.5]

12(h) Evaluate an indefinite or definite integral using a 'linear shift' substitution. (e.g. $\int x^2\sqrt{2x+3}\, dx$). [5.5]

13. Applications of Integrals

13(a) **(Optional)** Find the points of intersection of two curves, such as a line and parabola, two parabolas, or two functions of the form cx^n. [6.1]

13(b) **(Optional)** Find the area between curves using a definite integral. [6.1]

13(c) **(Optional)** Find the volume of a solid of revolution using disks or washers. [6.2]